

RESULTS FROM
FALL

Perfect

*Bohdan, Timothy E.	IN	Γ	'85
*Couillard, J. Gregory	IL	A	'89
*Gaston, Charles A.	PA	B	'61
*Gerken, Gary M.	CA	H	'11
*Gulian, Franklin J.	DE	A	'83
Gulian, Joseph D.			Son of member
*Harvey, Arthur J.	OH	A	'83
*Norris, Thomas G.	OK	A	'56
*Slegel, Timothy J.	PA	A	'80
*Weinstein, Stephen A.	NY	Γ	'96

Other

*Bannister, Kenneth A.	PA	B	'82
Barr, Robert A.	IL	E	'85
Coffman, John R.	MA	B	'66
Dechman, Don A.	TX	A	'57
Fogel, Arlene Beck	MA	Δ	'77
*Griggs Jr., James L.	OH	A	'56
Handley, Vernon K.	GA	A	'86
Johnson, Roger W.	MN	A	'79
Jordan, R. Jeffrey	OK	Γ	'00
Kistler, Aaron J.	TX	H	'90
Lalinsky, Mark A.	MI	Γ	'77
Leonard, Wyatt A.W.			member-relative
Marks, Lawrence B.	NY	I	'81
Marks, Benjamin			Son of member
Laxer, Teven			member-relative
*Melton, Walter C.	TX	A	'56
Parks, Christopher J.	NY	Γ	'82
Riedesel, Jeremy M.	OH	B	'96
Rogli, Victor L.	TX	Γ	'73
*Schmidt, V. Hugo	WA	B	'51
Sigillito, Vincent G.	MD	B	'58
*Spong, Robert N.	UT	A	'58
Spring, Gary S.	MA	Z	'82
Summerfield, Steven L.	MO	Γ	'85
Zison, Stanley W.	CA	Θ	'83

*Denotes correct bonus solution

FALL REVIEW

Regular Problem #2 (antimagic square) had 63 percent correct, #4 (pizza) had 74 percent correct, and the Bonus question (circles in sector) had 70 percent correct answers.

While 19 years is the most common difference between consecutive Blue Moons on New Year's Eve, it is not the average as some readers pointed out. One reader referred to a website that has results for 10,000 years and the average is 29.2 years. We accepted any of those answers.

WINTER ANSWERS

1 PROPS + KUDOS = HOWARD translates to **60369 + 74839 = 135208**, and LABOR + OF + LOVE + FOR + OVER = 60 + YEARS translates into **25768 + 61 + 2604 + 168 + 6048 = 60 + 34589** if YEARS is prime. This problem, like Howard's tenure as a judge, is a long one! Unfortunately, we don't have enough space to include it here. For those of our readers who are interested, the complete solution is available at www.tbp.org/pubs/brainTicklers.cfm.

2 Cate and Adam went to Doha, Dawn and Brad went to Elba, Emma and Chet went to Agra, Anne and Doug went to Bali, and Beth and Evan went to Cork. Reflecting on Cate's statement, it is clear that regardless of whether she or her husband is the liar, she will recount the opposite of what is true. Emma is, therefore, married to Chet. Since Chet could not have gone to Elba, Dawn is clearly a liar, and Beth was not in Doha. Because of Dawn's prevarications, we know Adam lies (and cannot be married to Dawn) and Beth tells the truth when she states that she is not married to Adam. Since Adam cannot be married to Anne, he must be married to Cate who must speak with veracity. Doug goes to Bali, and, therefore, cannot be married to Beth, leaving only Anne as a possibility for his betrothed. This leaves Beth married to Evan, and Dawn as Brad's bride. Evan is a liar, so Chet did not go to Doha after all. Chet must have gone to Agra and Cate is the only lady who could have gone to Doha. Adam, therefore, did not travel to Elba—Brad did, and Evan must have gone to Cork; this completes the pairings.

3 The latitude at which the sun is directly overhead at noon is $\arcsin(\sin(23^\circ)\sin((t-t_{VE})(360^\circ)/Y))$ degrees, where Y is the length

of an Earth year. If the sun is directly overhead a given point on the Earth's surface, the latitude of this point is nothing other than the declination of the sun. Consider three great circles: the equator, the ecliptic (which crosses the equator at an angle equal to the Earth's axis tilt), and the hour circle which passes through both poles and the image of the sun on the ecliptic. These three circles form a spherical triangle. Let the length of the side on the ecliptic be a and note its associated angle is $A = 90^\circ$. The length of the side on the hour circle is the latitude b , with associated angle $B = 23^\circ$. Using the law of sines for spherical triangles, we have $\sin b/\sin B = \sin a/\sin A$, which can be rearranged to give $\sin b = \sin a \sin B/\sin A$. Since a circular orbit is assumed, orbital speed is constant and, therefore, the length of a is directly proportional to the time since the sun transited the equator at the vernal equinox. As t_{VE} is known, we can write the proportionality as $a = (t-t_{VE})(360^\circ)/Y$ since we know the sun will move 360° along the ecliptic over the course of an Earth year Y . Plugging this in and solving for b gives us the final result shown above.

4 The wind speed was approximately **7.53 km/h**, and in calm air, the trip would take about **24.25 minutes**. We can write the expression of power exerted $P = Fv$, where F is the force used to overcome air drag. Since Griggs was able to cycle forward in spite of the headwind, and the tailwind has the same magnitude as the headwind, we need not concern ourselves with air *assistance*. As such, we know the force can be written as $F = k(v_{bike} - v_{wind})^2$, when we do not expect a negative F . If the morning trip of 10 km took 30 minutes, then $v_{AM} = 20$ km/h. The afternoon trip took 20 minutes, so $v_{PM} = 30$ km/h. Since power is constant, we can equate the power for the two one-way trips: $k(v_{AM} - v_{wind})^2 v_{AM} =$

$k(v_{PM} + v_{wind})^2 v_{PM}$. Cancelling k and plugging in the known values, and rearranging gives $(20 + v_{wind})/(30 - v_{wind}) = \sqrt{3/2}$. Solving for v_{wind} gives: $v_{wind} = (30\sqrt{3/2} - 20)/(1 + \sqrt{3/2}) = 130 - 50\sqrt{6} \approx 7.53$ km/h. The power exerted in calm air today is once again the same, so we can write: $k(v_{AM} - v_{wind})^2 v_{AM} = k(v_{today})^3$. Cancelling k and plugging in the known values once again gives $v_{today} = 10(750 - 300\sqrt{6})^{1/3} \approx 24.75$ km/h, and the time needed to complete a one-way trip is $10/v_{today} = (1440(5+2\sqrt{6}))^{1/3} \approx 24.25$ minutes.

5 The minimum number of crossings in the braid is six. To achieve this, consider holding the leather such that the long dimension is vertical, with the dark-stained side facing you. Grab the bottom right corner and curve it around to the back of the strip so the dark-stained side is facing away from you. Slide this corner through the right slit from behind, and subsequently slide it through the left slit from the front. While holding the top of the leather strip in a fixed position, adjust things so the corner you were holding has switched places with the bottom left corner, both of which now have the light side showing. At this point, the top left strip has a half twist and ends at the bottom right, while the top right strip has a half twist and ends at the bottom left, and the middle strip has three half twists. Now, grab the bottom right corner again, and curve it around the front of the strip so that the dark-stained side briefly faces you as you swing it through the bottom of the left slit from behind and through the bottom of the right slit from the front. When done, hold the top of the strip fixed and adjust things so that the corner you are holding becomes the bottom left corner with the dark-stained side facing you once again. At this point there will be no half twists in any of the three strips, and there will be a total of six crossings. Incidentally, this process can be repeated for long strips. Each set of the above moves will result in another six crossings as the braid grows.

Bonus. One solution for the circumferences of the two spheres is: **415,280,564,497/348,671,682,660** and **676,702,467,503/348,671,682,660**. The solution to the Bonus is rather complicated, and we don't have enough space to include it here. For those of our readers who are interested, the complete solution is available at www.tbp.org/pubs/brainTicklers.cfm.

Computer Bonus. For primes greater than 14,713 and less than 1 billion, there are **5 sequences** of thirteen prime numbers with a maximum gap of 8 between primes. The first primes in each sequence are 86,966,771, 172,039,573, 296,931,731, 369,008,657, and 875,753,231.

NEW SPRING PROBLEMS

1 I started with 10 cards with a different digit on each card. After discarding one card, I used the remaining nine cards to form a two-digit prime, a three-digit prime, and a four-digit prime (with no leading zeros). Even if I told you what the sum of these three primes was, you would not be able to deduce the digit I discarded; but if, in addition, I told you that the middle two digits of the sum were each equal to the digit I discarded, then you would be able to deduce the sum of the three primes I found. What was that sum?

—Susan Denhan in *New Scientist*

2 On each day of a non-leap year, I gave my daughter a new penny, which I obtained directly from the U.S. Mint and which I, consequently, assumed to be of standard weight. I learned, however, that there had been a minor production problem at the Mint one day during the year, and a few pennies had been released (I don't know on which day) that were heavier or lighter than standard. Assuming my daughter's penny collection contains at most one off-spec penny, what is the minimum number of weighings with a common two pan balance scale needed to determine whether or not she has an odd penny; and, if she

does, which is the odd penny and whether it is heavier or lighter than standard?

—The Puzzle Corner in *Technology Review*

3 Solve the following cubic cryptarithm. All the usual rules apply: the same letter always represents the same digit, each digit is represented by a unique letter, and there are no leading zeros.

$$ABCDE = (A + B + C + D + E)^3$$

—Source Unknown

4 Brag is a poker-like game played with a standard deck of 52 cards, but a hand consists of only three cards instead of the usual five. The possible hands that can be dealt are defined below:

- (1) Royal flush (AKQ in same suit)
- (2) Straight flush (sequence of 3 cards in same suit)
- (3) 3 of a kind (3 cards of same denomination)
- (4) Straight (sequence of 3 cards representing at least 2 different suits)
- (5) Flush (3 cards in same suit but not in sequence)
- (6) Pair (2 cards of same denomination plus an unmatched 3rd card)
- (7) High card (3 unmatched cards—2 or 3 different suits and 3 different denominations, not in sequence)

Aces can be high or low for a straight. How many of each type of hand are there? Present your answer as a table with 7 rows and 2 columns. Enter the numbers 1 to 7 in column 1 (no need to repeat the whole definitions); in column 2 list the number of different ways of being dealt each type of hand. Be careful not to double count. For example, if you count all the straight flushes, you will be counting the Royal Flushes twice.

—*The Ultimate Book of Card Games*
by The Diagram Group

5 What is the remainder when the number $2^{4,700,063,497}$ is divided by 4,700,063,497?

—*Prime Numbers, the Most Mysterious Figures in Math*
by David Wells

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BRAIN TICKLERS

Continued from page 31.

Bonus. Consider a chain (loop or circle) of length $L = n(n+1)/2$ with links of integral lengths 1, 2, 3, 4, ..., n , occurring sequentially. You desire to tightly wrap the chain around three posts that form the vertices of an equilateral triangle with vertices a distance of $S = L/3$ apart, so that the posts occur exactly at a break between two links. The smallest example of such a chain has $n = 9$ links; for this chain, $S = L/3 = 9(10)/6 = 15$; one post is between links 3 and 4, the second post is between links 6 and 7, and the third post is between links 8 and 9, so the three sides are $(4 + 5 + 6)$, $(7 + 8)$, and $(9 + 1 + 2 + 3)$. What is the largest chain with less than 1,000 links that will form an equilateral triangle? What is the shortest chain that can be wrapped around 3 posts in more than one way? Make the usual simplifying assumptions: the chain and the corner posts have zero radii, there is no overlap where the ends of the chain join, etc.

— The Puzzle Corner in
Technology Review

Computer Bonus. Find three consecutive triangular numbers whose sum is a perfect square. The first such set is $15 + 21 + 28 = 64 = 8^2$. What is the third such set? A triangular number is a number of the form $n(n + 1)/2$, where n is a positive integer.

—*Elementary Number Theory*
by David M. Burton

Postal mail your answers (the method of solution is not necessary) to any or all of the Brain Ticklers to **Dylan Lane, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697** or email to BrainTicklers@tbp.org as plain text only. The cutoff date for entries to the Spring column is the appearance of the Summer *Bent* which typically arrives in mid-June. (The digital version is a few days earlier). We welcome any interesting problems that might be suitable for the column. The Computer Bonus/Double Bonus is not graded. Dylan will forward your entries to the judges who are **F.J. Tydeman, CA Δ '73**; **J.C. Rasbold, OH A '83**; **J.R. Stribling, CA A '92**; and the columnist for this issue,

— **H.G. McIlvried III, PA Γ '53**

TWO CHAPTER PETITIONS RECEIVED

Two petitions for new collegiate chapters of Tau Beta Pi have been received.

The Tau Engineering Honor Society has operated at Georgia Southern University since 2017. The institution is a public research university with a flagship campus in Statesboro. Founded in 1906 as a land grant college, Georgia Southern is the fifth largest institution in the University System of Georgia and is the largest center of higher education within the southern half of Georgia. Georgia Southern is classified as a Doctoral and Research Institution and consists of eight primary colleges.

In 2010, Georgia Southern received approval to offer three new engineering degrees: bachelor of science in civil engineering, bachelor of science in electrical engineering, and the bachelor of science in mechanical engineering.

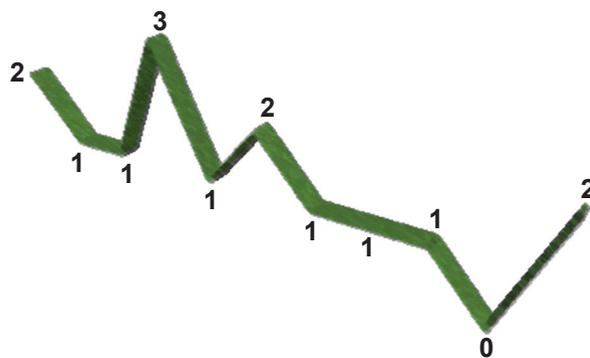


Chart above is the number of new chartered chapters per year of Tau Beta Pi in the last eleven years (2010 thru 2020).

As of March 2020, the total number of TBPI collegiate chapters will be 256, with six inactive chapters.

Petitioning chapters must be approved by the Executive Council and the Convention. Georgia Southern would be TBPI GA Gamma Chapter and Merrimack College would be TBPI MA Iota Chapter.

The Merrimack College of Engineering Honor Society has operated at Merrimack College since December 13, 2017. The institution is a selective, private college in Boston, Massachusetts. It was founded in 1947 in North Andover, Massachusetts, by the Order of St. Augustine with “an initial goal to educate World War II veterans.”

The college features more than 100 career-focused undergraduate, professional, and graduate programs. Three engineering bachelor’s degrees are offered in civil, electrical, and mechanical.

The campus encompasses 220 acres and 40 buildings with the library named after the founder of the college, Rev. Vincent A. McQuade.

WINTER ANSWERS

Solution to Tickler No. 1 in the Winter 2020 Issue of The Bent. This Tickler is most easily solved by writing a computer program. However, with a little patience and tenacity, it can be solved without computer aid. One approach is as follows: Define $c_1, c_2, c_3,$ and c_4 as the carry bits for the tens, hundreds, thousands, and ten thousands columns, respectively. Note that $H=1,$ and that $O \neq 0,$ since this would either lead to $P=R$ or $D=A.$ Start by considering successively larger values for $S,$ starting with $S=2.$ This defines $D,$ and we can then exhaustively cycle through values of $O,$ starting with $O=2$ (or $O=3,$ since here $S=2$) using the equation $O+D+c_2=A+10c_3$ to find one of two values for $A.$ Since $P+K+c_4=10+O,$ we have only two possible values for $P+K,$ and the choice of P will establish c_2 by use of the equation $P+O+c_1=R+10c_3,$ and confirm a viable A (if any). For every viable P and K pair, this additionally identifies a value for $R.$ This leaves two unused digits to try and map to U and W through the equation $R+U+c_3=W+10c_4.$ Following this procedure carefully reveals that no value of S from 2 to 8 provides a valid solution; and, therefore, $S=9$ and $D=8.$ Trying $O=2$ forces $A=0$ with $c_2=1.$ $P+K=11$ or $12,$ so either $P=7$ and $K=4$ or $P=7$ and $K=5.$ Either choice gives $R=0=A,$ which is invalid. Trying $O=4$ gives $A=2$ with $c_2=0$ or $A=3$ and $c_2=1.$ Since $P+K=13$ or $14,$ no value of P or K will work to give $A=2$ with $c_2=0.$ For $A=3$ and $c_2=1,$ either $P=6$ and $K=7$ which leads to an invalid $R=1,$ or $P=7$ and $K=6$ which leads to $R=2.$ While this looks promising, there is unfortunately no way to map the remaining values of 0 and 5 to U and $V.$ Trying $O=5$ gives $A=3$ with $c_2=0$ or $A=4$ and $c_2=1.$ Since $P+K=14$ or $15,$ no value of P or K will work at all. This will be true for any $O>5$ as well, so $O=3$ leading to $A=2$ with $c_2=1.$ $P+K=12$ or $13,$ so there are four possibilities: $P=5$ and $K=7$ which leads to an invalid $R=9;$ $P=7$ and $K=5$ or $P=7$ and $K=6,$ both of which lead to an invalid $R=1.$ Therefore, $P=6$ and $K=7,$ leading to $R=0.$ The remaining digits can be mapped to U and V such that $U=4$ and $V=5.$ The second problem is even more challenging as there are 15 possible solutions before considering the constraint on YEARS being prime. As such, it lends itself well to computer assistance, and with a little effort one can arrive at $V=0, F=1, L=2, Y=3, E=4, A=5, O=6, B=7, R=8$ and $S=9,$ leading to $PROPS + KUDOS = HOWARD$ translates into $60369 + 74839 = 135208,$ and $LABOR + OF + LOVE + FOR + OVER = 60 + YEARS$ translates into $25768 + 61 + 2604 + 168 + 6048 = 60 + 34589$ if YEARS is prime.

WINTER ANSWERS

Solution to Bonus Tickler in the Winter 2020 Issue of the The Bent. Start with the formula for the volume of a sphere, We must find two new diameters, x and $y,$ such that $(4/3)\pi(x/(2\pi))^3 + (4/3)\pi(x/(2\pi))^3 = (4/3)\pi(2/(2\pi))^3 + (4/3)\pi(1/(2\pi))^3.$ Cancelling the constant terms gives the simple relation $x^3 + y^3 = 23 + 13 = 9.$ The goal is to find rational solutions to this curve. Since we have one rational solution already [namely (2,1)], we can use the chord-tangent method to find other rational solutions on the curve. The theory behind this method is that a tangent line to the curve at a known rational point will have a double rational root; and, therefore, any other intersection of this line with the curve must necessarily be rational as well. In fact, if any line intersects this curve at two points known to be rational, any potential third intersecting point will again be rational. Our situation is shown in the adjacent figure. The red curve is a graph of $x^3 + y^3 = 9,$ and point A is our known rational solution (2,1). We can construct a tangent line to our curve at point A through implicit differentiation: $y=(9-x^3)^{1/3}$ so $dy=(9-x^3)^{-2/3}(-x^2)dx.$ At $x=2, dy/dx=-4,$ so the tangent line (shown in green) is $y-1=-4(x-2)$ or $y=-4x+9.$ Plugging this value of y into the original curve and multiplying out gives $-63x^3+432x^2-972x+720=0.$ Factoring out the double root at $(x-2)^2$ gives a root at $x=20/7,$ so $y=-17/7,$ shown as point B in the graph. While this is a rational solution to our curve, y is regrettably negative, so it cannot be a solution for the sphere circumference. We could repeat the above process until we find a solution in the first quadrant. However, this approach will rapidly produce rational fractions with giant numerators and denominators, so another approach is needed. Note that the point B', which is symmetric with B about the line $y=x,$ is obviously rational $(-17/7,20/7).$ The blue line passing through A and B' also crosses through point C. We have argued above that C must be a rational solution. A little math shows that the equation of the blue line is $y=(-13/31)x+57/31.$ Plugging this value of y into the original curve and multiplying out gives $27,594x^3+28,899x^2-126,711x-82,926=0.$ Factoring out the roots known from points A and B' $(x-2)(7x+17)$ gives a root at $x=-271/438,$ so $y=919/438.$ Alas, we are still not in the first quadrant with this solution at C, so, creating a tangent line to the curve at point C (shown in purple) will intersect the curve at our desired solution S in the first quadrant. Some careful bookkeeping reveals that the equation for this purple tangent line is $y=(-73,441/844,561)x+1,726,596/844,561.$ Plugging this into our curve and multiplying out, factoring out the double root $(x+271/438)^2$ will give our desired result $x=415,280,564,497/348,671,682,660$ and $y=676,702,467,503/348,671,682,660.$

